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Phase Combined Systems

The purpose of this paper is to identify some of the issues, which are inherent to phase combined systems and to describe them in quantifiable terms such as to allow for intelligent decisions with regards to system specifications and manufacturing processes.

The example used for this purpose involves a simple two way phase combined system consisting of two amplifiers, two waveguide lengths and one 90 degree quadrature hybrid.

Figure A is used to examine the nature of the hybrid.

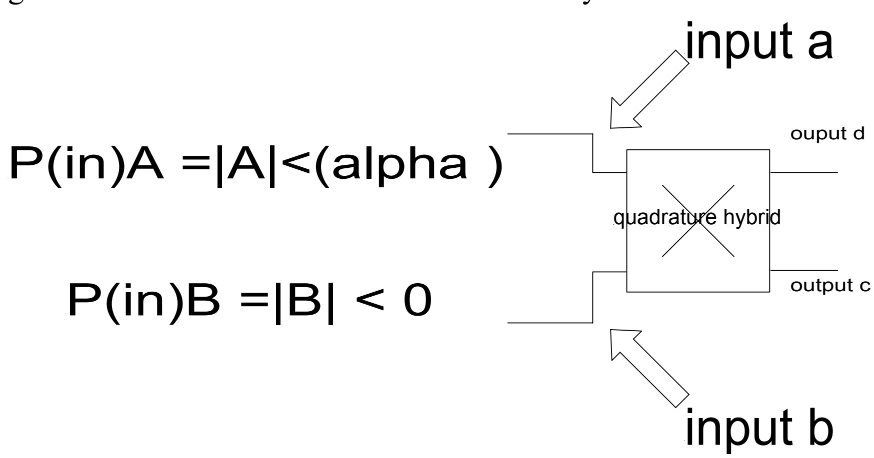

Figure A Quadrature Hybrid Phase Combining

E-Field interaction in the hybrid must be considered when deriving the required equations.

Recall: Power = E^2/Z.

The problem can be addressed as a vector algebra equation.

$$\text{Output d (Watts)} = [\mathrm{sqrt}(A/2)\cos(\propto°) + j\,\mathrm{sqrt}(A/2)\sin(\propto°) + \mathrm{sqrt}(B/2)\cos(-90°) + j\,\mathrm{sqrt}(B/2)\sin(-90°)\,]\,\hat{}\,2$$

$$= [\mathrm{sqrt}(A/2)\cos(\propto°) + j\,\mathrm{sqrt}(A/2)\sin(\propto°) - j\,\mathrm{sqrt}(B/2)]\,\hat{}\,2$$

AMPLITUDE OUTPUT d = sqrt[(A/2 (cos $\propto$)^2 +((sqrt(A/2)sin($\propto$°) -sqrt(B/2))^2]^2
Phase angle output d = Tan^{-1} [(sqrtA/2sin($\propto$ rad)-sqrt(B/2)) / (sqrt(A/2)cos($\propto$ rad))]

Output c can be derived in a similar fashion and will result in the compliment response of output d.

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Figure B shows the normalized output d power where input power A is equal to input power B in amplitude and frequency while the phase relationship is varied.

Figure C shows the normalized output power where the phase angle alpha is held at –90 degrees and the input power ratio B/A is varied. In the case where input power B is twice that of A, given that A = 1 W, B=2 W, then the total available power is 3W. The plot indicats that 97 % will appear at port d, or 2.91 W. Port c will have 0.09 W.

Figure B. Phase Sensitivity

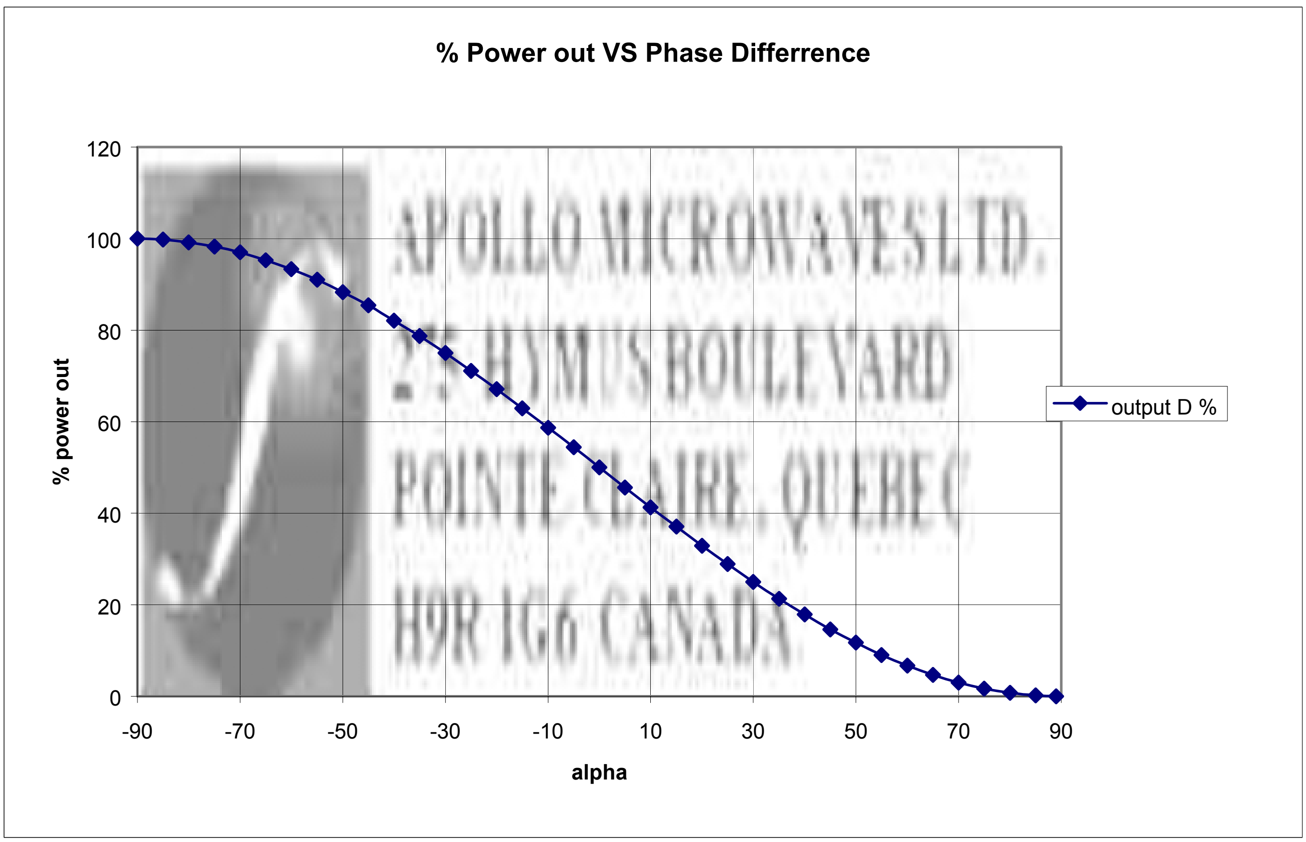

Figure D shows how the phase of the combine signal varies with the phase variation of the input signal.

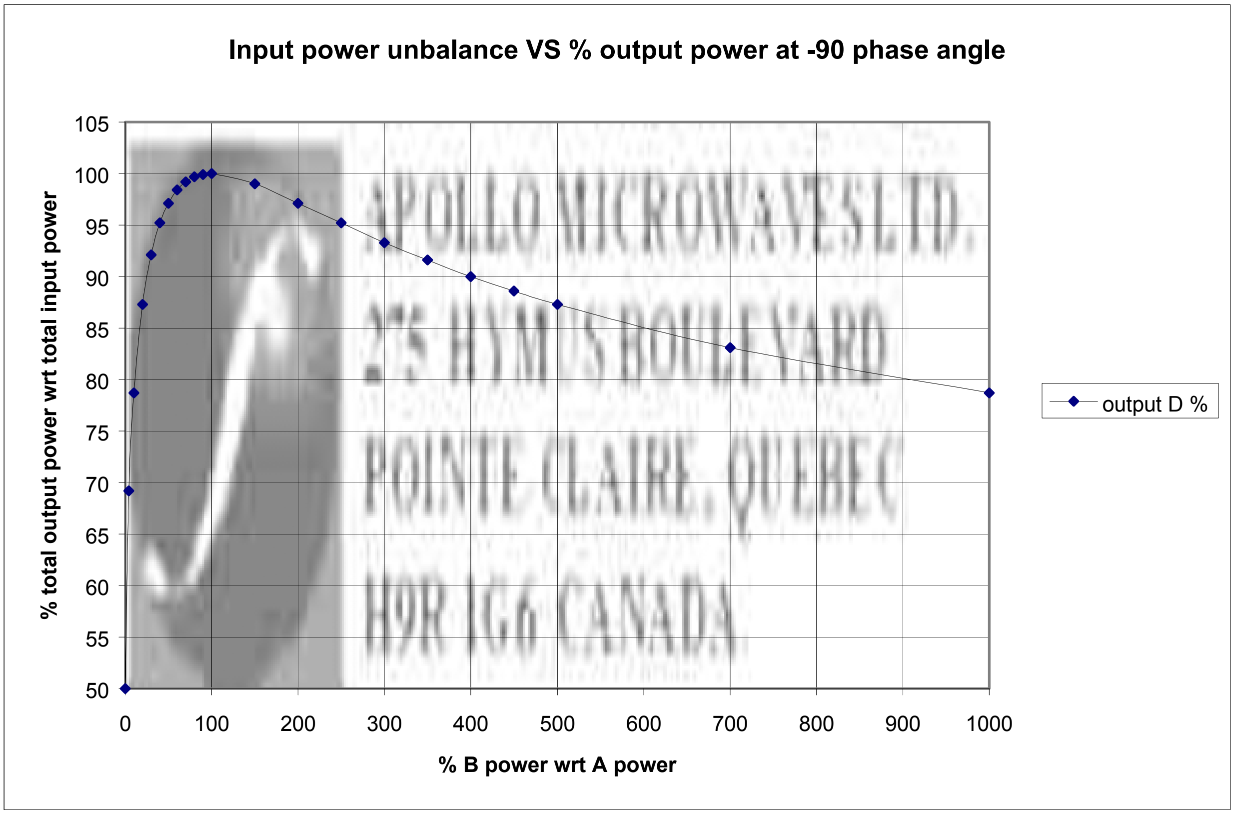

Figure C Normalized Output Variation to Input Power Imbalance

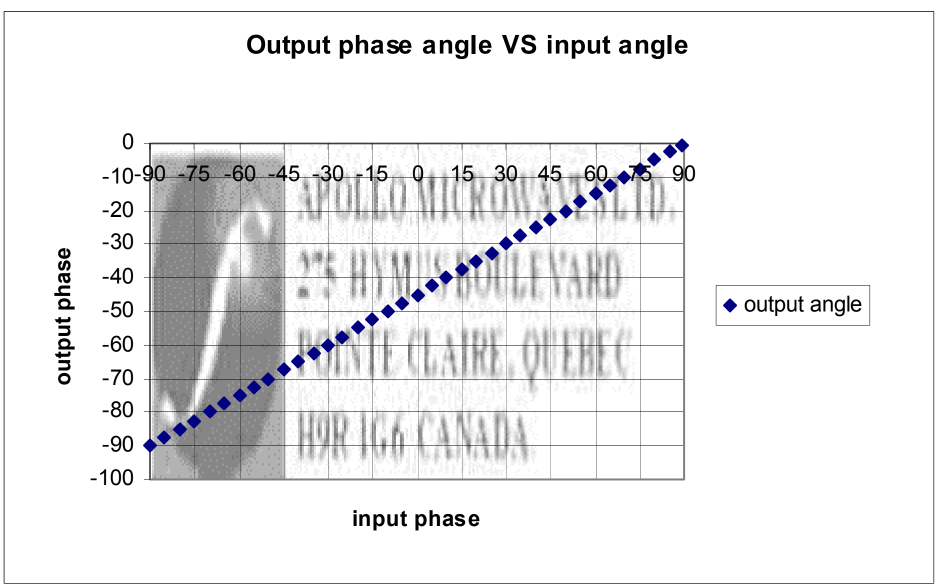

Figure D. Output Phase relationship with Respect to input phase angle.

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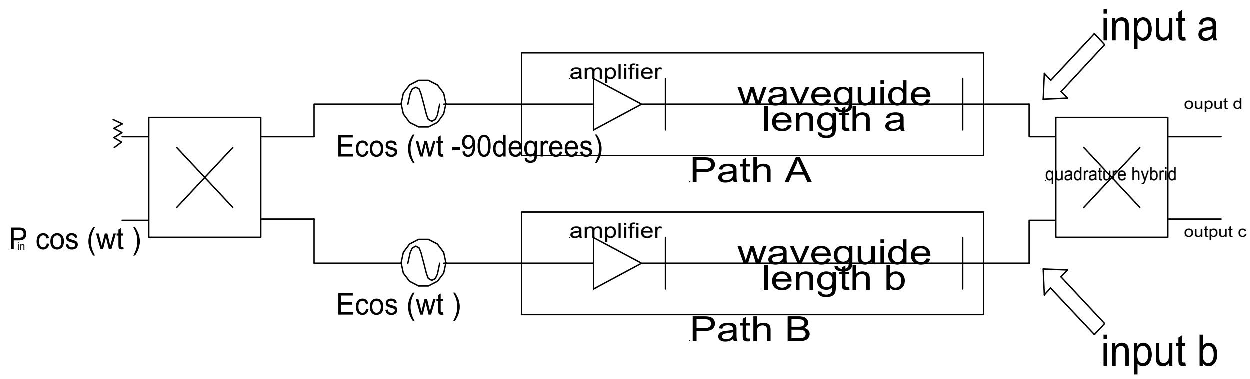

Figure 2 Phase Combined System

The phase relationship of interest is that of (input a) and (input b). The ideal relationship for the configuration shown in figure 1, for the output port d, implies that the phase angle of (input a) lags the phase angle of (input b) by exactly 90 degrees.

Assume that the gain (A) and electrical length of the amplifier in path A is equal to that of path B.

In the first scenario referring to figure 1, given that the spatial length of path A is equal to the spatial length of path B, then the phase relationship of the signal at input a with respect to the input Ecos (wt - 90°) can be predicted.

Similarly, the phase relationship of the signal at input b with respect to the input Ecos (wt) can be predicted.

Units for phase will be degrees for the comparison.
One wavelength represents 360 degrees.
Waveguide wavelength (λg) is in inches. Recall that (λg) is frequency dependant.
Spatial Length of the waveguide paths are in inches.

Number of wavelengths in the path (Npath) = spatial length of the waveguide / λg.

Path phase = Npath * 360°

In the example shown in figure 1, the phase difference between path A and path B is of interest.
Ideally the difference would be zero. A difference of any 360 degree cycle will have the same results as a zero difference in phase, therefore integer multiples of 360 degrees are also acceptable.

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This situation applies for any given frequency or waveguide wavelength. The problem arises due to the frequency dependency of waveguide wavelength.

The following exercise will demonstrate the point.

This example will assume WR 28 waveguide for figure 1.

Lets say that the waveguide length for both path a and path b are equal and are 64.03 inches.

If the frequency applied is 28.0 Ghz, then the wavelength (λg) into WR28 waveguide is 0.6403 inches.

(Npath) = 64.03 inches / 0.6403 inches
= 100.00
*Path A phase = 100.00 * 360° = 36 000°*
*Path B phase = 100.00 * 360° = 36 000°*

The signal at (input a) is Ecos (wt -90degrees) delayed by 36 000 degrees. Or Ecos (wt) < -36090°.

The signal at (input b) is Ecos (wt) delayed by 36 000 degrees. Or, Ecos (wt) < -36000°.

The difference in phase between input a and input b is –90 degrees.

If the frequency applied is 30.0 Ghz, then the wavelength (λg) into WR28 waveguide is 0.5529 inches.

(Npath) = 64.03 inches / 0.5529 inches
= 115.81
*Path A phase = 115.81 * 360° = 41 690°*
*Path B phase = 115.81 * 360° = 41 690°*

The signal at (input a) is Ecos (wt -90degrees) delayed by 41 690° . Or Ecos (wt) < -41 780°.

The signal at (input b) is Ecos (wt) delayed by 41 690°. Or, Ecos (wt) < -41 690°.

The difference in phase between input a and input b is –90 degrees.

Thus, we conclude that when the electrical delays of both paths are identical, there is no frequency dispersion effects due to waveguide lengths.

Next, we explore the effect of having unequal paths.

Now, we can repeat the example with different lengths (path a will be shorter than path b).
Again, path A is 64.03 inches, Path B is 64.6703 (conveniently 1 wavelength longer at 28 GHz).

If the frequency applied is 28.0 Ghz, then the wavelength (λg) into WR28 waveguide is 0.6403 inches.

(Npath A) = 64.03 inches / 0.6403 inches
= 100.00
(Npath B) = 64.6703 inches / 0.6403 inches
= 101.00

*Path A phase = 100.00 * 360° = 36 000°*
*Path B phase = 101.00 * 360° = 36 360°*

The signal at (input a) is Ecos (wt -90degrees) delayed by 36 000 degrees. Or Ecos (wt) < -36 090°.

The signal at (input b) is Ecos (wt) delayed by 36 360 degrees. Or, Ecos (wt) < -36 360°.

The difference in phase between input a and input b is +270 degrees. Or -90 °.

If the frequency applied is 30.0 Ghz, then the wavelength (λg) into WR28 waveguide is 0.5529 inches.

(Npath A) = 64.03 inches / 0. 5529 inches
= 115.81
(Npath B) = 64.6703 inches / 0. 5529 inches
=116.97
(the difference in wavelengths between the two paths at 30.0 GHz is 1.16, as opposed to 1.0 for 28 GHz)

*Path A phase = 115.81 * 360° = 41691.1°*

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*Path B phase = 116.97 * 360° = 42107.6°*

The signal at (input a) is Ecos (wt -90degrees) delayed by 41691.1 degrees. Or Ecos (wt) < -41781.1°.

The signal at (input b) is Ecos (wt) delayed by 42107.6 degrees. Or, Ecos (wt) < -42 107.6°.

The difference in phase between input a and input b is +416.5 degrees.
416.5 is divisible by 360 by 1.16 times. Removing the full 1*360 degree cycles leaves a remainder of 0.16 * 360° = +57.6°.(unacceptable for a phase combined system)
Reminder: the ideal difference is -90° and a practical difference is 90°+/- 25°

If the frequency applied is 26.0 Ghz, then the wavelength (λg) into WR28 waveguide is 0.7753 inches.

(Npath A) = 64.03 inches / 0. 7753 inches
= 82.59
(Npath B) = 64.6703 inches / 0. 7753 inches
=83.41
(the difference in wavelengths between the two paths at 30.0 GHz is 1.16, as opposed to 1.0 for 28 GHz)

Path A phase = 82.59 360° = 29732.4°*

*Path B phase = 83.41 * 360° = 30027.6°*

The signal at (input a) is Ecos (wt -90degrees) delayed by 29732.4 degrees. Or Ecos (wt) < -29832.4°.

The signal at (input b) is Ecos (wt) delayed by 30027.6 degrees. Or, Ecos (wt) < -30027.6°.

The difference in phase between input a and input b is +195.5 degrees.
(again, unacceptable for a phase combined system)

A better approach to the problem is to first define a length tolerance, say, around one wavelength of differential error as the acceptable tolerance for the system.

Next we define an acceptable phase error based on system performance criteria. For example, for 95 % output, or -0.22 dB insertionloss variation, we can permit phase dispersion of around +/- 25° based on the attached phase variation versus output power plot.

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Then, based on this, we can determine what the useable bandwidth of the system will be about a given center frequency. Take 28 GHz as the center frequency.

We can greatly simplify the calculations by recognizing that at the center frequency, given the one wavelength maximum error that the phase difference of the system in figure 1 is +270° as previously shown.

The problem is finding which frequencies will result in a phase error of no more than +295° and no less than +245°. The allowable variation in phase error is +/- 7 %.
The solutions begin with the equations;

> *(Npath B)- (Npath A)*= 64.6703 inches /λg1- 64.03 inches /λg1 = *0.93*
> *0.93* *λg1=0.6403 (the number of wavelengths of path difference times the center frequency wavelength)
> λg1=0.69" : frequency approximately 27.1 Ghz
>
> *And*
> *(Npath B)- (Npath A)*= 64.6703 inches /λg2- 64.03 inches /λg2 =*1.07.*
> *1.07**λg2=0.6403.
> λg2 = 0.59 " : frequency approximately 28.9 Ghz

Thus, we have shown that for WR28 waveguide, a center frequency of 28 Ghz, a permissible insertion loss variation of +/- 0.22 dB, and a path imbalance of 0.6403 inches, we have a useable bandwidth of 1.8 GHz, or around 6.4 %.

Figure 2 demonstrates the calculated results. In fact, all the calculations shown can be avoided by using a plot such as this in conjunction with the permissible phase variation (Also extracted from a plot of output sensitivity to phase error).

The same conditions applied to WR34 waveguide in place of the WR28 waveguide is shown in Figure 3. Table 1 contains the data for both charts. The items highlighted in red show the range of +/- 25 degrees about 28 GHz.

Of interest, is the fact that WR34 waveguide shows a slightly lower slope in the response. In Wr34, for this example, we get a bandwidth improvement of 200 MHz for the same performance, yielding a bandwidth of around 7%.

Figure 4 demonstrates two points.
1. By changing the lengths from 64 inch ranges to 10 inch ranges, and maintaining the same absolute difference in length of 0.64 inches between path a and path b, it is evident that only the path difference plays a role in the performance due to phase, and not the overall lengths of the paths.
2. By reducing the difference in path lengths, the slope of the phase variation is reduced, thus improving performance. A length error of 0.16 inches, or approximately ¼ λg, the overall performance can be met from 26 GHz to 31 GHz.

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Figure 5 shows the same relationship in WR34, and also shows performance improvement.

The performance discrepancy between WR28 and WR34 waveguide reduces as the spatial error converges toward zero.

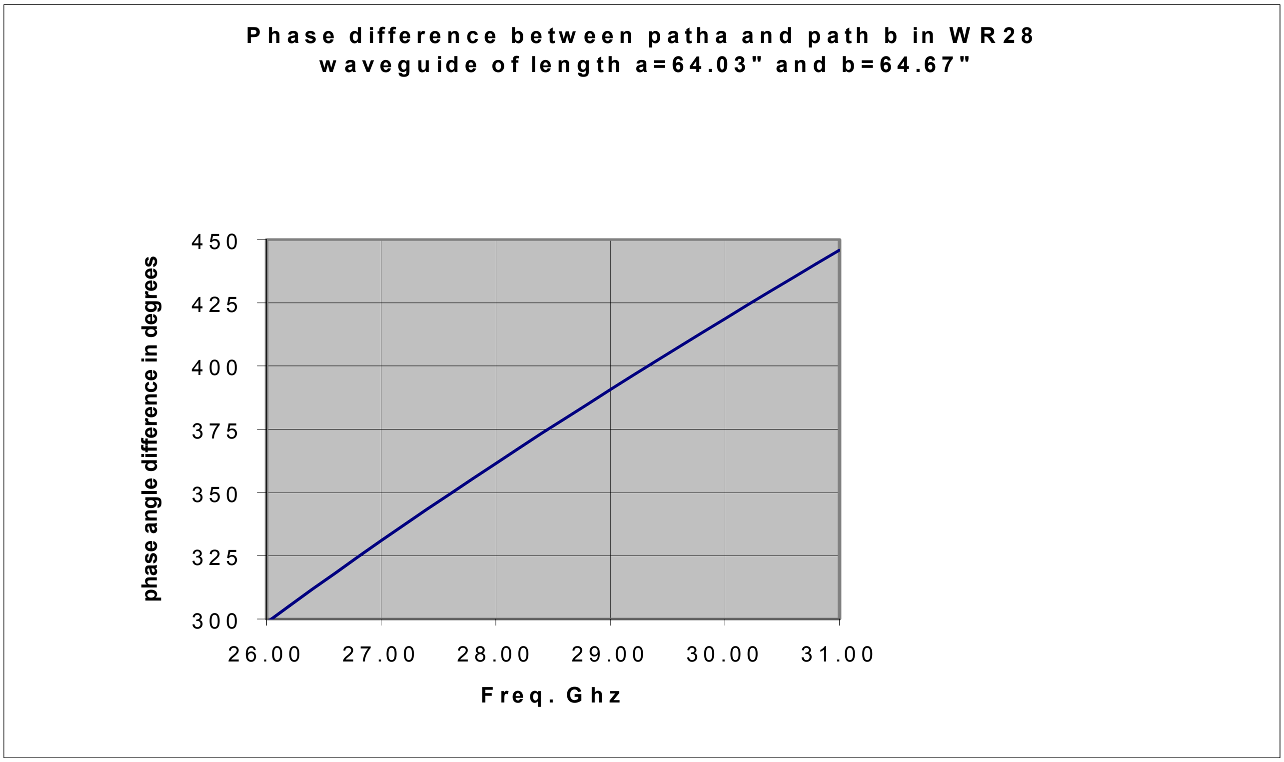

Figure 3 Phase Angle Sensitivity of Wr28 with 0.64" difference

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Table 1: Chart Data

Frequency GHz	Difference in delta lgs * 360 degrees for WR28	Difference in delta lgs * 360 degrees for WR34
26.00	299	380
26.20	305	385
26.40	312	390
26.60	318	395
26.80	325	400
27.00	331	406
27.20	337	411
27.40	343	416
27.60	349	421
27.80	356	426
28.00	361	431
28.20	367	436
28.40	373	441
28.60	379	446
28.80	385	451
29.00	391	456
29.20	396	461
29.40	402	465
29.60	408	470
29.80	413	475
30.00	419	480
30.20	424	485
30.40	430	489
30.60	435	494
30.80	440	499
31.00	446	504

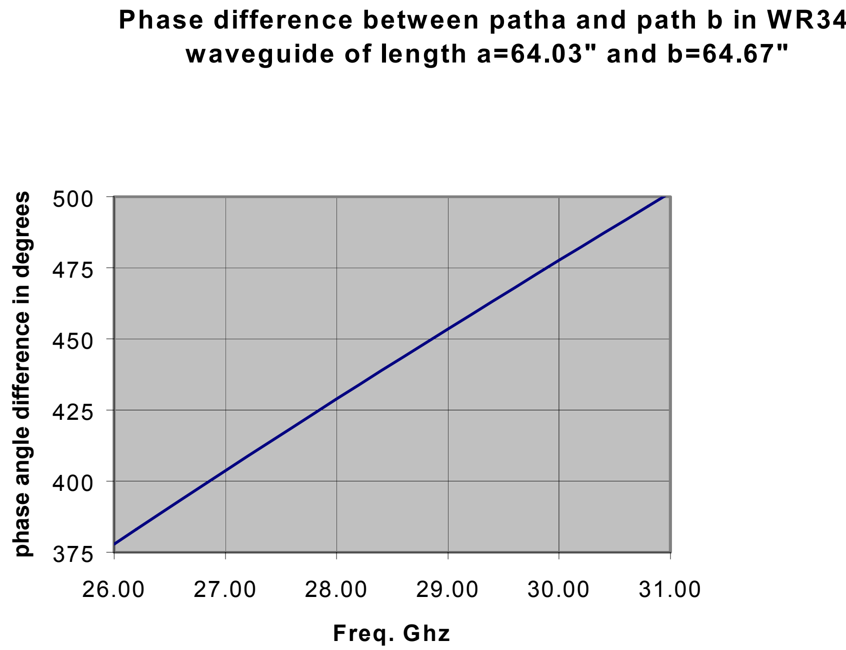

Figure 4 Phase Angle Sensitivity of Wr34 with 0.64" difference

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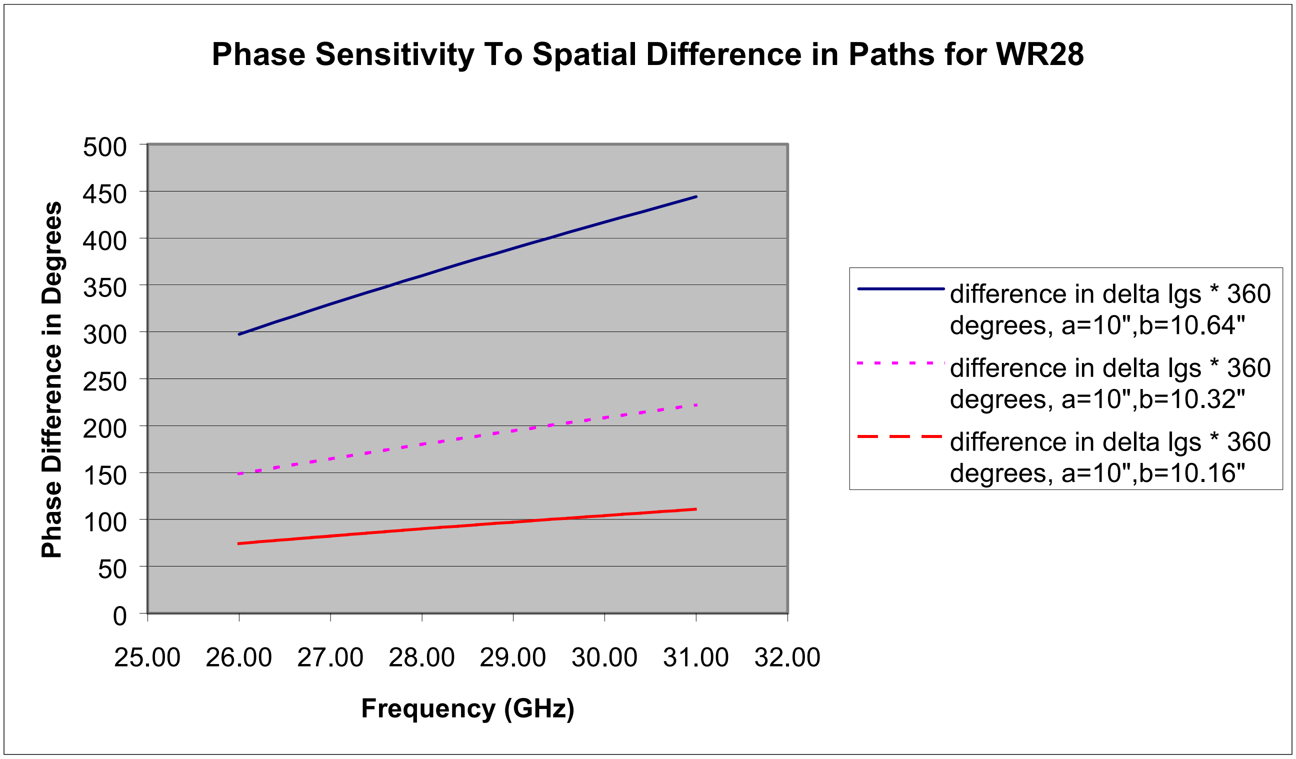

Figure 5 Comparison of Dispersion for Different Path Errors in WR28

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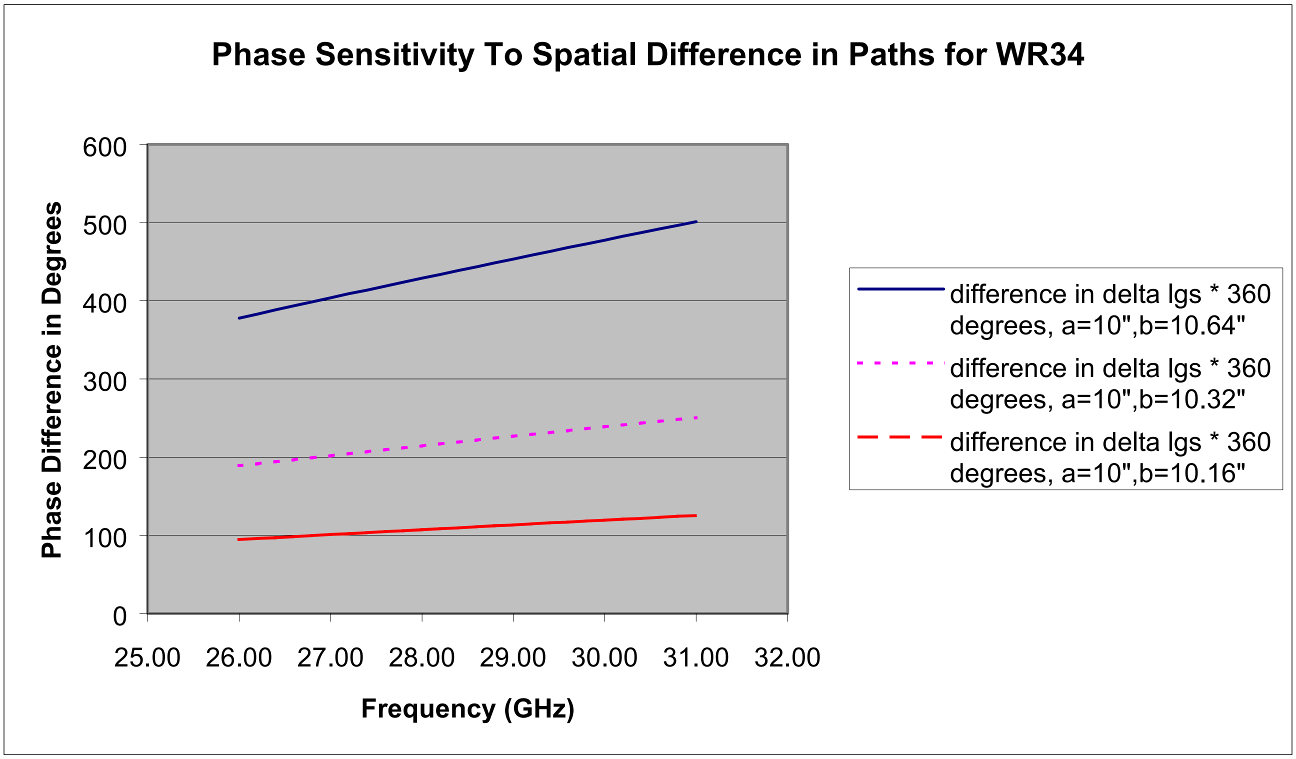

Figure 6 Comparison of Dispersion for Different Path Errors in WR34

Conclusions:

In phase-combined systems, the sensitivity to phase error about the optimal phase angle difference is relatively low. Practical combined systems can tolerate phase angle deviations of up to +/- 25 degrees while maintaining output level variations within three tenths of a dB.

Sensitivity to input power imbalance is negligible in practical terms.

The dispersive nature of waveguide wavelength is also predictable and quantifiable. In a given system, the phase response over a band of frequencies is easily determined and can be used in response analysis.

Because the tolerance to phase deviation and the associated effect is predictable and quantifiable, a system level performance can be predicted using the phase response over the desired frequency range.

The fact that phase relation between two signals traveling through different paths is a differential quantity allows for the nulling effect of temperature effects.

If that were not the case, in the example of WR28 waveguide that is ten inches in length and made of copper, it can be shown that if one path were 100 degrees Celsius higher than the other (a highly unlikely scenario since both paths are in the same vicinity and are subjected to same conditions), the overall effect in phase difference of the two paths is manageable. ($\Delta L = \alpha * \Delta T$,for copper α = 16.74 E-6 inch/inch, for a rise of 100 C the length increase would be 16.74E-4 inches for every inch of length. Or 0.017 inches overall , 10 degrees in phase error).

The system design is reduced to determining the acceptable performance criteria and the associated manufacturing tolerances. Bandwidth ratios of 7% with +/- 0.3 dB output level deviations can be achieved with path errors of around one wavelength which may be achievable without individual tuning from system to system for dispersion effects.

Greater bandwidths are possible, but require tighter tolerances or individual length adjustments.

These parameters can be easily determined and managed by maintaining a close relationship between the various sub-components in the system design.

Apollo Microwaves Ltd. has a wide and varied library of phase combined systems operating in the field. These systems range from S-band to Ka Band and are configured as 2-way, 3-way and 4-way system.

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Appendix A

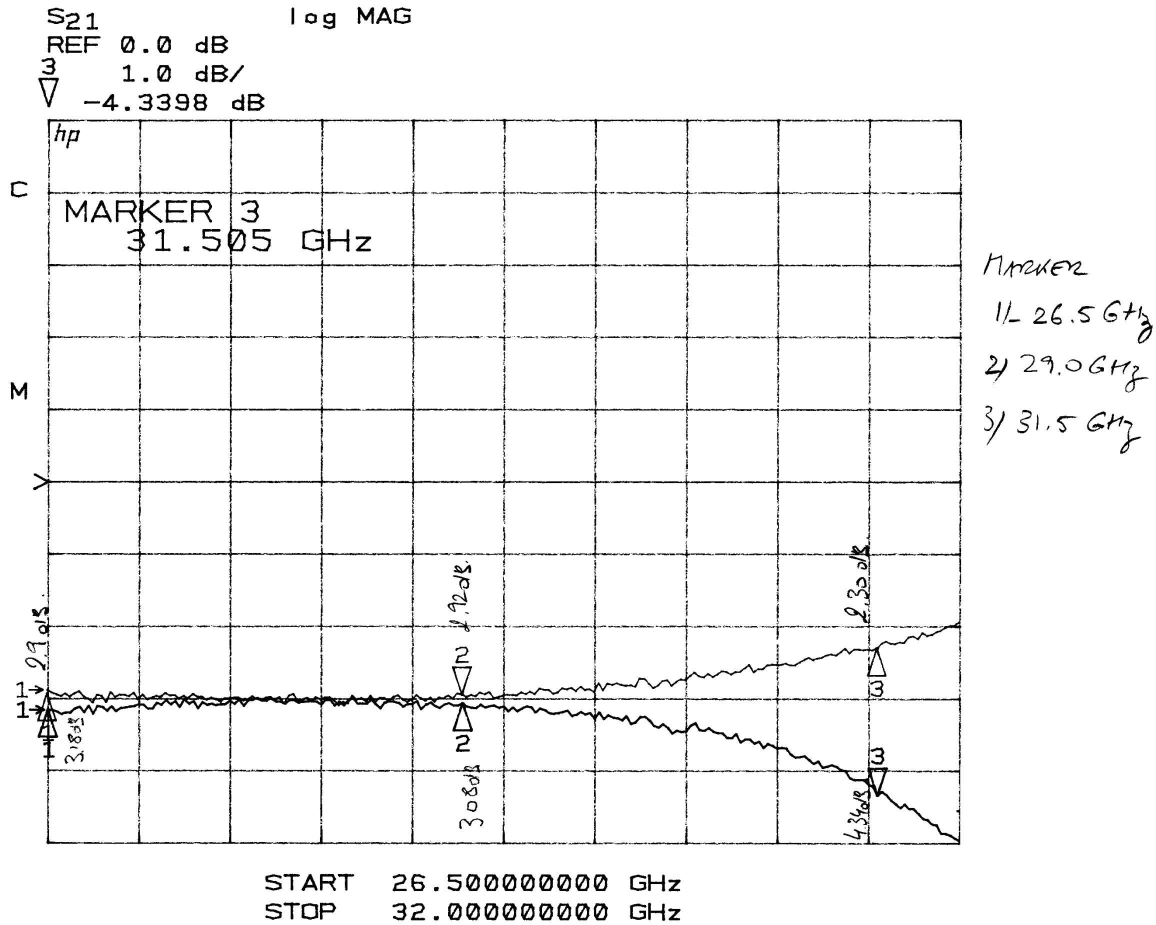

Figure 7 Quadrature Hybrid Insertion Loss Plot (port c and port d)

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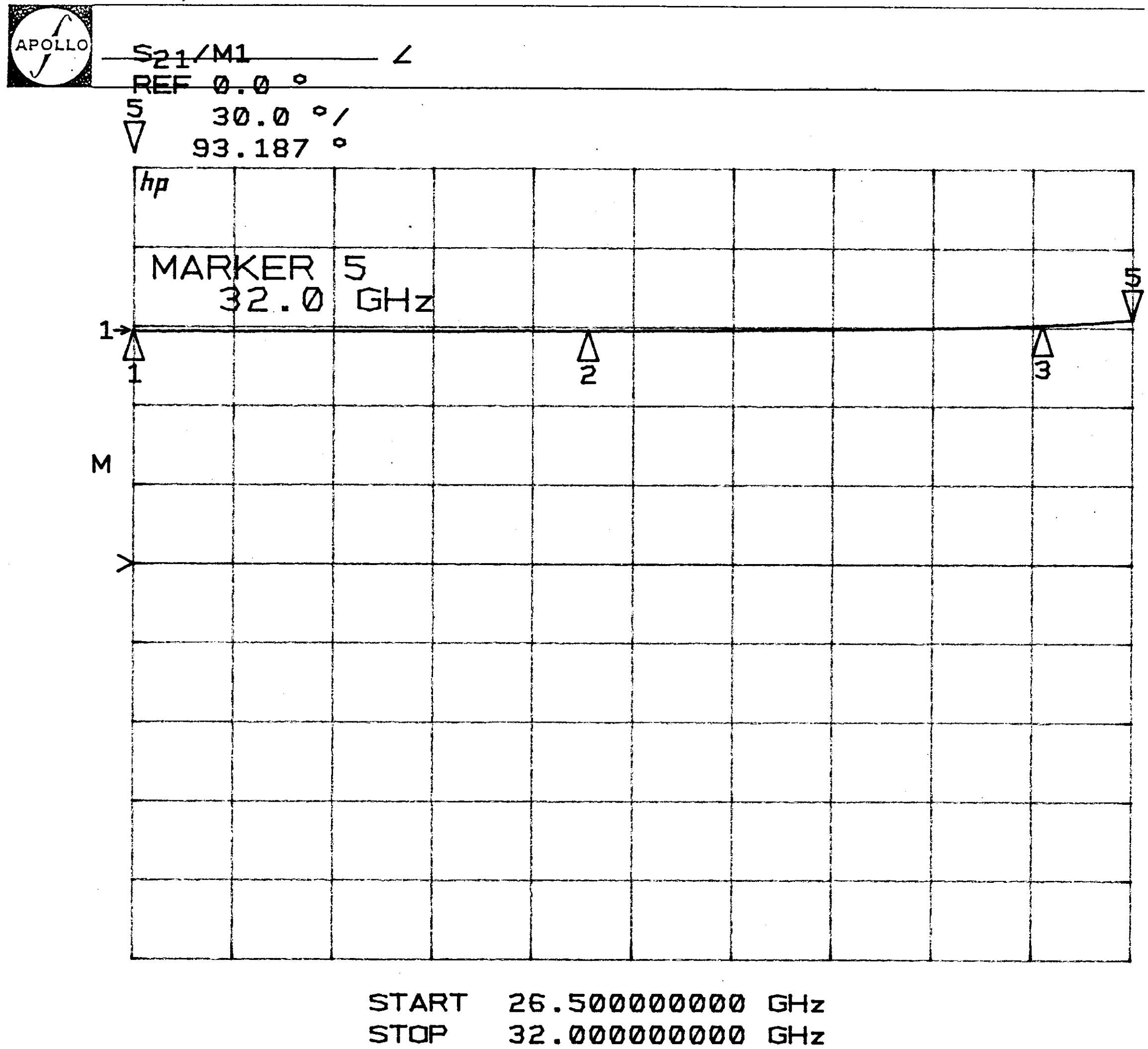

Figure 8 Quadrature Hybrid Phase Response Empirical Plot

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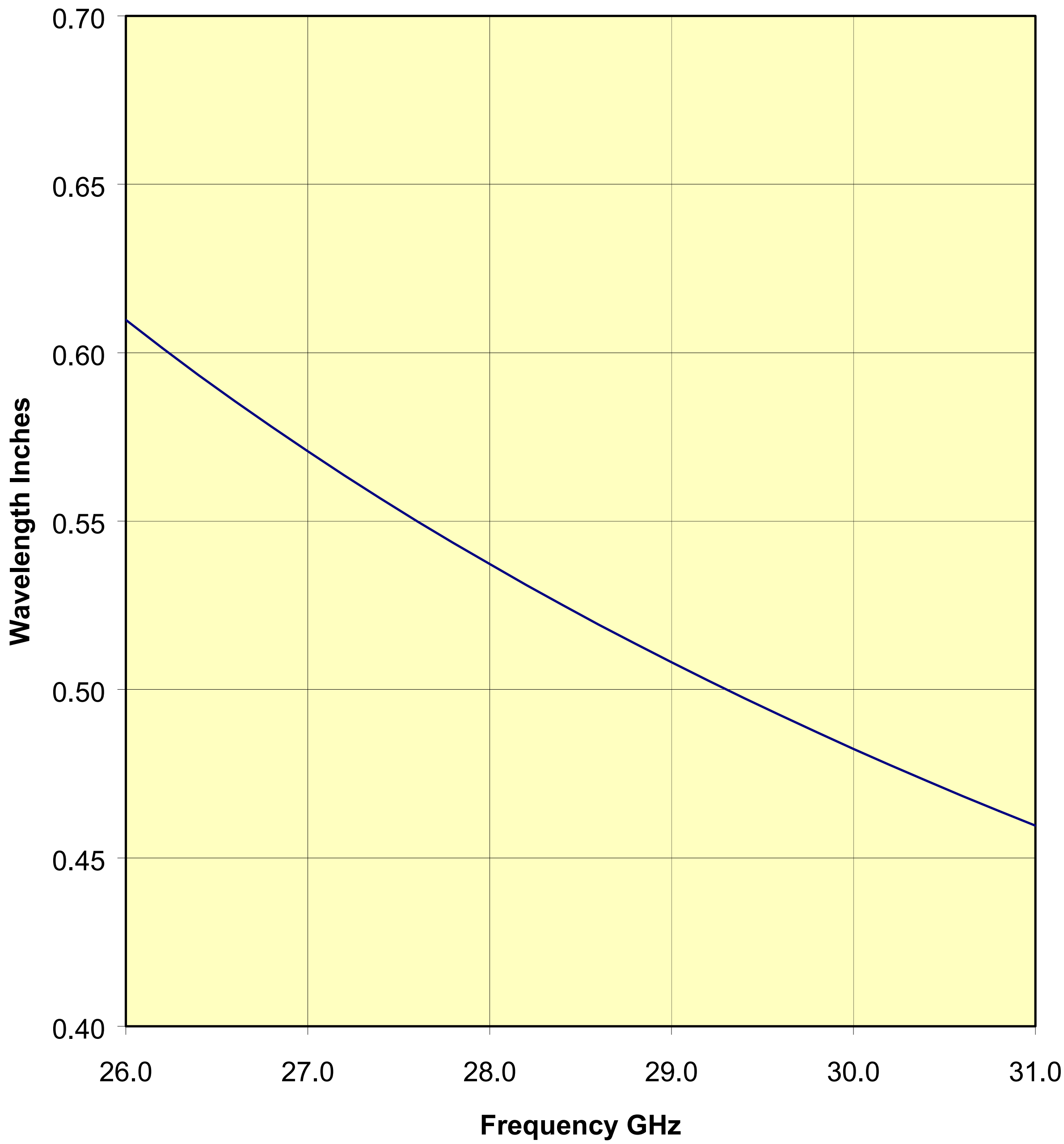
Waveguide Wavelength VS Frequency For WR34 Waveguide



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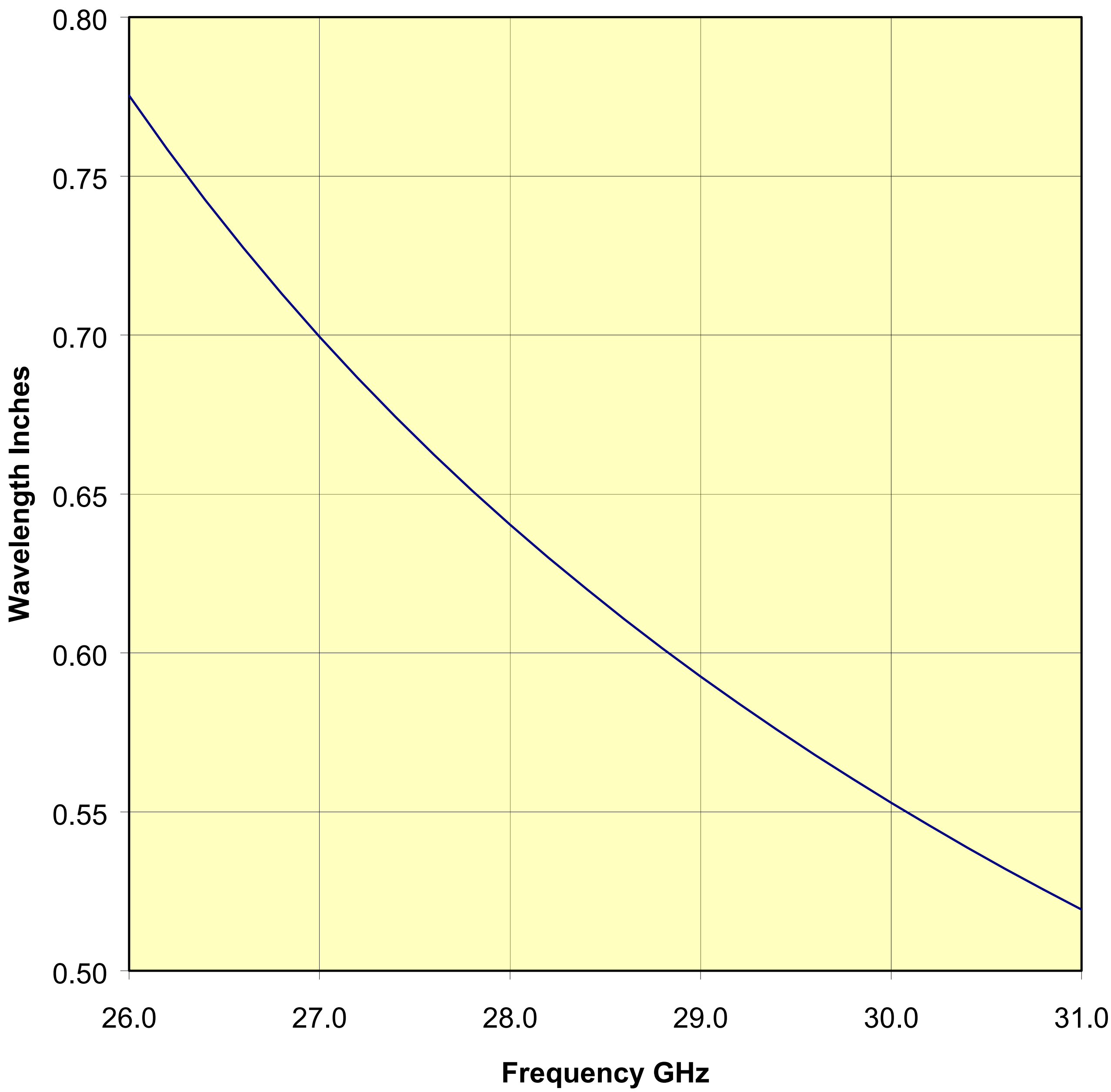
Waveguide Wavelength VS Frequency For WR28 Waveguide



Delta wavelength/5GHz for WR34 = 0.150 inch
Delta wavelength/5GHz for WR28 = 0.256 inch

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Practical Considerations

There are various approaches that can be taken to realize a phase-combined system. The most common approaches include hybrid, magic tee and VPC (Variable Power Combiner) as the phase combining medium.

Hybrid and magic tee approaches have the advantage of being mechanically simpler. However, they do not provide automatic redundancy without the addition of a switching network. With a switching network, a failed chain would result in an immediate 6 dB reduction in output power. This would be followed by an interruption in the order of 100 mseconds while bypass switching occurs. The end result is one online chain and 3 dB less output power than in the phase combined mode.

The VPC approach allows phase combining with inherent "soft fail" redundancy. As with the hybrid and magic tee approaches, in the case where one chain fails, there is an immediate 6 dB reduction in output power. However, in the VPC approach, this event is followed by VPC switching without interruption of output power. Within 150 mseconds of the event, the output power will be 3 dB less than in the phase-combined mode. The VPC configuration has a clear advantage where any interruption in output signal is not acceptable, such as in applications where synchronizations can be lost.